Improved Method for Determining Vehicle and Occupant Kinematics in Full-Scale Crash Tests

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This paper presents details of proposed improvements to the ways that full-scale crash test data are analyzed. In particular, the basic formulation of the flail space method presented in NCHRP Report 230 is shown to contain several simplifications that have never before been examined in detail. The NCHRP Report 230 formulation and its history are briefly reviewed, followed by an extensive discussion of improvements that would make the flail space method more physically correct and more general. The potential sources and magnitudes of error in the current model are explored, and new techniques are described. Examples using actual crash test data are provided to illustrate the typical magnitudes of error associated with each simplification. A computer program is briefly described that implements these improvements.

When mathematical models are used to approximate physical phenomena, errors tend to arise because some effects are neglected. The challenge in building mathematical models is to include the most important effects while neglecting those that only add effort to the analysis. This paper investigates the flail space model, which calculates the hypothetical occupant impact velocity with a vehicle interior in a roadside appurtenance collision. The original model proposed in NCHRP Report 230 includes the most important effects for many crash test scenarios. The purpose of this paper is to investigate some additional effects that have been generally neglected and to determine their importance in terms of the fidelity of the flail space model.

The concepts of the flail space method and the occupant risk criteria were introduced in 1981 by Michie in NCHRP Report 230 (1). This method has become the standard means of evaluating the potential hazard to vehicle occupants involved in an impact with a roadside appurtenance. Prior to the publication of NCHRP Report 230, several evaluation criteria were used based on the particular device under consideration: change in momentum for small sign supports and luminaires, 50-msec average accelerations for longitudinal barriers, and maximum average deceleration for crash cushions. Perhaps the greatest advantage in using the occupant risk criterion is that it unifies a variety of evaluation criteria into easily calculated quantities.

Shortly after NCHRP Report 230 was published, some researchers began to improve the basic flail space model. In 1983, Calcote et al. developed a program to calculate the occupant risk parameters that accounted for the yaw rotation of the vehicle (2). A data reduction program was modified that had been in use at Southwest Research Institute since the early 1970s. This program calculated the vehicle kinematics by numerically integrating the coupled equations of motion. Calcote et al. added procedures to measure and compare the position and velocity of the occupant with respect to the passenger compartment. This program was an improvement to the NCHRP Report 230 approach because it (a) used coupled equations of motion, and (b) more realistically modeled the occupant's trajectory through the passenger compartment.

In a 1987 review of NCHRP Report 230, Ray et al. noted that many injuries in longitudinal barrier collisions occurred when a nonimpact side occupant travels across the passenger compartment and strikes the vehicle interior on the impact side (3, 4). This can increase the flail space five-fold. In another paper, the same authors show that most of the severe injuries occur after the initial occupant impact (5). The NCHRP Report 230 formulation and the Calcote model both cease analysis after the first occupant contact. Ray et al. suggest that events after the initial occupant impact may be more important although the flail space method, as currently formulated, only applies to the first occupant/interior collision.

Recently, other authors have noted the importance of some of these same improvements. For example, Ross et al. have improved the Highway-Vehicle-Object-Simulation Model (HVO-SM) by adding an occupant risk model that tracks the position of the occupant after the initial occupant/passenger compartment impact (6). They also use Ray et al.'s observation that nonimpact side and out-of-position occupants have greater flail distances, thus increasing the occupant risk values. This effect was confirmed in crash tests of small cars impacting concrete safety-shape barriers. Their procedure appears to be the same as Calcote's, with the exception that calculations continue until both the side wall and the instrument panel have been struck or the run has terminated. Owings has numerically integrated the two-dimensional coupled equations of motion to determine vehicle motion after impact with a small sign support (7).

Although the NCHRP Report 230 formulation of the flail space method is an improvement over the multiplicity of evaluation criteria previously used, it neglects some significant physical effects. The importance or negligibility of these effects has never been published, and the magnitudes of the potential errors are unknown. The purpose of this paper is to

- Investigate several neglected effects.
- Assess importance of these effects in terms of the accuracy of the resulting data.
THE NCHRP REPORT 230 FORMULATION OF THE FLAIL SPACE MODEL

The occupant impact velocity defined by NCHRP Report 230 (1) is the hypothetical impact velocity of the vehicle occupant with the passenger compartment surface (8). This value is calculated by operating on transducer data obtained in full-scale crash tests. The occupant velocity is found when the occupant, acting as a free missile, has traveled either 1 ft laterally or 2 ft longitudinally. The time of impact is found using the following expression:

\[ r = \int_{t_0}^{t_f} a_s \, dt \, dt \]  

\[ s = \int_{t_0}^{t_f} a_l \, dt \, dt \]  

where

\[ r = \text{displacement in the vehicle longitudinal direction.} \]
\[ s = \text{displacement in the vehicle lateral direction.} \]
\[ a_s = \text{acceleration in the vehicle longitudinal direction.} \]
\[ a_l = \text{acceleration in the vehicle lateral direction.} \]
\[ t_0 = \text{time of vehicle impact with appurtenance.} \]
\[ t_f = \text{time when the occupant has traveled the longitudinal flail distance, and} \]
\[ t_1 = \text{time when the occupant has traveled the lateral flail distance.} \]

The accelerations \( a_s \) and \( a_l \) are the values measured by the transducers mounted to the vehicle during the impact event. The values of \( r \) and \( s \) are recommended in NCHRP Report 230 as 2 ft and 1 ft, respectively. Equations 1 and 2 can be solved for times of impact \( t_0 \) and \( t_f \). When these times have been identified, the occupant impact velocity can be calculated using the following equations:

\[ v_{\text{occ-r}} = \int_{t_0}^{t_f} a_s \, dt \]  

\[ v_{\text{occ-l}} = \int_{t_0}^{t_f} a_l \, dt \]  

where

\[ v_{\text{occ-r}} = \text{occupant impact velocity in the vehicle longitudinal direction.} \]
\[ v_{\text{occ-l}} = \text{occupant impact velocity in the vehicle lateral direction.} \]

After time \( t_0 \) and \( t_f \), the critical evaluation parameter is the ridedown acceleration—the value of the highest 10-msec average acceleration in each of the principal directions. Calculating the ridedown accelerations involves finding the largest value of the 10-msec average acceleration in each direction. The formulation of the flail space model given in Equations 1 through 4 is easy to use and can be accomplished using either graphical, manual, or computer methods.

Several important assumptions are implicit in the NCHRP Report 230 formulation of the flail space model. The model does not account for the effect of the rotation of the vehicle on the occupant impact velocity, but treats the longitudinal and lateral directions independently. It ignores the out-of-plane displacements and velocities, and it assumes the transducers measure the response of the vehicle center of gravity. These assumptions make the calculation of the occupant impact velocity straightforward, although it is unclear what effect they might have on errors in the calculated values. Some currently used data reduction programs simply apply Equations 1 through 4 directly, while others make some attempt to account for these effects. A presentation of these issues and a discussion of their importance, however, has never appeared in highway safety literature. The following sections present a discussion of several of the most notable implicit assumptions and the associated errors that result in calculating the occupant risk parameters.

SENSITIVITY TO TRANSDUCER POSITION

In Equations 1 through 4, the accelerations are presumed to be measured at the vehicle center of gravity. In most typical crash tests, it is not possible to place the transducers exactly at the center of gravity because of uncertainty about the true position, as well as the geometric constraints of the vehicle. When the occupant and vehicle kinematics are calculated using acceleration data, which was not measured at the center of gravity, errors are introduced into the computations. It would be useful to know how the transducers should be located in full-scale crash tests.

There is no reason to prefer measurements made at the center of gravity over those made at other positions on the vehicle. By convention, the velocities of the center of gravity are used to represent the kinematics of the entire vehicle. The following discussion pertains to cases where two linear transducers and one rate gyro are used to measure the vehicle longitudinal and lateral acceleration as well as the yaw rate. The choice of this transducer arrangement implies that the vehicle motion is expected to be primarily planar. Other transducer arrangements will also have the potential for errors due to mislocation, and similar steps can be taken to minimize these errors.

Derivation

Figure 1 shows a vehicle where the transducers are not located at the center of gravity. The quantities \( p_r \) and \( p_l \) are the longitudinal and lateral distances from the center of gravity to the position of the transducers. From elementary dynamics (9), the acceleration in the longitudinal direction, \( r \), of a point away from the center of gravity can be expressed as

\[ a_r = a_{r,c} - p_r \ddot{r} + p_l \ddot{\theta}, \]
Since the acceleration of the point is known and the acceleration at the center of gravity is unknown, Equation 5 can be solved for \(a_{rs}\):
\[
a_{rs} = a_{r} + p\dot{\theta}; + p\ddot{\theta},
\]
(6)
A similar equation for the lateral, s, direction can be derived in the form
\[
a_{ss} = a_{s} + p\dot{\theta}; - p\ddot{\theta},
\]
(7)
The difference in sign between Equations 6 and 7 is due to the sense of the angular velocity. The maximum difference between the acceleration measured at an arbitrary point and measured at the center of gravity can be found by rearranging those equations:
\[
|a_{rs} - a_{r}| = |p\dot{\theta}; + p\ddot{\theta}|,
\]
(8)
\[
|a_{ss} - a_{s}| = |p\dot{\theta}; - p\ddot{\theta}|.
\]
(9)
Since angular acceleration, \(\dot{\theta}\), is not typically measured in a crash test, it must be estimated in Equations 8 and 9 using the angular velocity. Angular velocity is often measured using rate gyros. Figure 2 contains a plot of typical yaw rate gyro data. Since the rate gyro data is gathered at discrete time intervals, direct information about the derivative of the angular velocity, \(\ddot{\theta}\), is lost. In addition, the rate gyro data, like all transducer data, contains high frequency noise resulting from vehicle vibrations and electronic signal noise. Data such as that shown in Figure 2 cannot be used directly to obtain information about the derivative of the angular velocity.

It is common practice to integrate data obtained by accelerometers to gain the velocities and displacements. This is generally accurate since integrating tends to smooth a curve and the derivative (the experimental data) includes all the information required to define the function, except the initial conditions. Thus, velocities are obtained by integrating the acceleration curve once, displacements are obtained by integrating accelerations twice, and the heading angle is obtained by integrating the yaw velocity once.

However, differentiating raw transducer data presents a different problem. When a continuous function is estimated by sampling discrete points along a curve, information about the derivative, or slope, of the function is degraded. There are several alternatives to dealing with this problem:

- A smooth curve can be fitted using a variety of numerical techniques with the experimental data to estimate the actual response. The derivatives of this curve can then be obtained at the points of interest and used in Equations 8 and 9. The difficulty with this approach is that the solution cannot proceed stepwise through the data; the rate gyro data must be smoothed before the acceleration data can be reduced. The dark curve in Figure 2 represents a curve fitted to the experimental data. This technique is the preferred method of estimating the angular acceleration.
• If the interval of time between discrete samples is small in comparison to the smallest frequency apparent in the experimental data, the derivative of the angular velocity can be approximated by calculating the velocity's change during the time interval. This is the definition of the derivative if the time interval is taken to be infinitely small. The question of how small the time step should be to avoid errors is not easily answered. It is unclear what order of magnitude of errors might be associated with using this technique.

Regardless of the approach, if $\dot{\theta}$ is taken to be adequately smoothed rate gyro data, the following procedure can be used where the subscripts $t$ are used to denote a particular time step and $\Delta t$ to denote the size of the time step used in acquiring the data. Equations 6 and 7 can be rewritten as follows:

\[
a_{c,xt} = a_{c,xt} + p_x \dot{\theta}_x \Delta t + p_z \dot{\theta}_z \Delta t \\
\]

\[
a_{c,yt} = a_{c,yt} + p_y \dot{\theta}_y \Delta t + p_z \dot{\theta}_z \Delta t \\
\]

Perhaps the best solution for eliminating the problem of differentiating angular velocity data is to use rate gyro’s in the transducer package. Some agencies involved with full-scale crash testing use two sets of three accelerometers (longitudinal, lateral, and vertical directions) placed at known locations on the vehicle. Since there are six degrees of freedom DOFs (motion in each of the three directions and rotation about each of the three axes) and six accelerometers, a system of coupled equations can be obtained that, when solved, provide the three translational accelerations and three angular velocities directly. The six coupled equations can be found in a variety of texts and reports dealing with the dynamics of rigid bodies (10, 11). Unfortunately, this technique is also sensitive to errors in measuring the positions of the transducers accurately although it is less sensitive to measurement errors than when using rate gyro’s.

Typical Errors in Full-Scale Crash Tests

If worst case values of $p_x = 1\, \text{ft}$, $\dot{\theta}_x = 1.5\, \text{rad/sec}$ (86\°/sec) and $p_y = 30.0\, \text{rad/sec}$ are assumed, it is apparent from Equation 8 that the measurement of the transducers will be in error 32.2 ft/sec\(^2\) or 1 g. In a typical redirectional crash test, a vehicle experiences accelerations generally less than 10 g so the error associated with misplacing the transducers as much as 1 ft away from the center of gravity could be approximately 10 percent.

The expressions above can be used to calculate the accelerations of the center of gravity given the position of the transducers on the test vehicle and the acceleration histories. Placing the transducers as accurately as possible and adjusting measurements taken at points away from the center of gravity is good practice. It is difficult, however, to determine the actual position of the center of gravity on a test vehicle. The above analysis indicates that, if the position of the vehicle center of gravity were erroneously calculated, a significant error is possible. It is reasonable to assume that the transducers can be placed consistently within 12 in. of the vehicle center of gravity; whether they can be placed within 6 in. or 1 in., however, is unclear.

### COUPLED EQUATIONS OF MOTION

The NCHRP Report 230 formulation neglects the fact that the components of vehicle velocity are coupled. Equations 1 through 4 contain the assumption that the longitudinal velocity has no effect on the lateral velocity and vice versa. No lateral velocity or acceleration terms appear in Equations 1 and 3, and, likewise, no longitudinal velocity or acceleration terms appear in Equations 2 and 4. The velocities are coupled and neglecting this effect can cause large errors. The general equations of motion can be derived from elementary dynamics as demonstrated in the following paragraphs (12, 13).

#### Derivation

Figure 3 shows a vector, $V$, which is both rotating and changing its magnitude in the global $x$-$y$-$z$ coordinate frame. The orientation and magnitude of the vector are both functions of time. There is another coordinate reference frame denoted by the subscripts $r$-$s$-$t$ that rotates with respect to the $x$-$y$-$z$ coordinate system. In the case of the flail space model, the vector $V$ is the velocity vector of the vehicle, the $r$-$s$-$t$ coordinate axes represent the reference frame attached rigidly to the test vehicle, and the $x$-$y$-$z$ reference frame describes the global coordinate system attached to the earth. It might be expected that the rate of change of $V$ would be different when viewed from the vehicle-fixed $r$-$s$-$t$ reference frame than when viewed from the global $x$-$y$-$z$ reference frame. If $i$, $j$, and $k$ are unit vectors in the $r$-$s$-$t$ reference frame, the vector $V$ can be expressed as

\[
V = V_i i + V_j j + V_k k
\]

Differentiating $V$ with respect to time yields the rate of change of $V$:

\[
V_{\dot{t}} = \dot{V}_i i + \dot{V}_j j + \dot{V}_k k
\]

To find the rate of change of $V$ in the global $x$-$y$-$z$ reference frame, it should be noted that the unit vectors $i$, $j$, and $k$ are also functions of time when differentiating Equation 12. The rate of change in the global coordinate system is given by

\[
V_{\dot{t}} = \dot{V}_s s + \dot{V}_c c + \dot{V}_r r + \frac{V_{si}}{\dot{t}} i + \frac{V_{sj}}{\dot{t}} j + \frac{V_{sk}}{\dot{t}} k
\]

The first three terms of Equation 14 are the components of the rate of change of $V$ in the $r$-$s$-$t$ reference frame as shown.

![FIGURE 3 Vector in two coordinate reference frames.](image-url)
in Equation 13. The partial derivatives represent the angular velocity of the r-s-t reference frame. If a vector $\Omega$ is defined as the angular velocity of the r-s-t frame, the last three terms of Equation 14 are the cross product of $\Omega$ with $V$. Using this fact and substituting Equation 13 into 14 yields

$$\ddot{V}_{rs} = \dot{V}_{rs} + \Omega \times V$$

(15)

where $V_{rs}$ is the acceleration vector in the global coordinate system. It is these accelerations that are measured by the transducers fixed to the vehicle during a crash test. The yaw component of $\Omega$ is measured by the yaw rate gyro in a typical crash test. If Equation 15 is resolved into its component parts in the vehicle-fixed r-s-t coordinate system, the following system of three coupled linear differential equations is obtained.

$$a_r = \dot{v}_r + v\dot{\theta}_p - v\dot{\theta}_r$$

(16)

$$a_s = \dot{v}_s + v\dot{\theta}_r - v\dot{\theta}_s$$

(17)

$$a_t = \dot{v}_t + v\dot{\theta}_s - v\dot{\theta}_t$$

(18)

where the $r$, $s$, and $t$ subscripts on velocities and accelerations indicate the longitudinal, lateral, and vertical directions of the vehicle. The $r$, $p$, and $y$ subscripts on the angular velocities represent the roll, pitch, and yaw angles as shown in Figure 4. Equations 16 through 18 can be rearranged so the quantities measured in a crash test all appear on the right-hand side:

$$\dot{v}_r = a_r - v\dot{\theta}_p + v\dot{\theta}_r$$

(19)

$$\dot{v}_s = a_s - v\dot{\theta}_r + v\dot{\theta}_s$$

(20)

$$\dot{v}_t = a_t - v\dot{\theta}_s + v\dot{\theta}_t$$

(21)

In most crash testing applications, the roll and pitch rates and the vertical acceleration are generally small. The yaw rate, $\dot{\theta}_r$, and the lateral and longitudinal accelerations are usually the predominant variables measured in full-scale crash tests. All DOFs have been included above for completeness.

Since the system of equations is coupled, values for the components of velocity must be determined by solving the system simultaneously. There are several techniques for accomplishing this, although the fourth-order Runge-Kutta method is probably the most direct. This method is well suited to this type of data analysis since it proceeds in a stepwise manner using only the state of the system during the previous time step and the measurements for the current step. This allows the data reduction to proceed from the beginning of the impact event through the end with no prior knowledge regarding the exit conditions of the vehicle. Details of the numerical solution of coupled differential equations can be found elsewhere (14, 15).

**Accuracy of the Numerical Solution of the Coupled Equations of Motion**

To demonstrate the highly accurate nature of the numerical integration scheme used, consider the exact solution to the following problem:

Given

$$a_r = a \sin at$$

$$\dot{\theta}_r = \text{some constant}$$

$$a_s = b \sin at$$

$$v_{rs} = E$$

$$a_t = 0$$

$$v_{rt} = 0$$

Equations 16 through 18 become

$$a_r = \dot{v}_r - v\dot{\theta}_r$$

(22)

$$a_s = \dot{v}_s + v\dot{\theta}_s$$

(23)

$$a_t = 0$$

(24)

Equations 22 and 23 can be uncoupled by first differentiating every term in both equations with respect to time, yielding

$$\ddot{a}_r = \ddot{v}_r - \dot{\theta}_{r}\dot{v}_r$$

(25)

$$\ddot{a}_s = \ddot{v}_s + \dot{\theta}_s\dot{v}_s$$

(26)

Solving Equation 23 for $\dot{v}_s$ and substituting into Equation 25 gives

$$\dot{v}_s + \ddot{\theta}_s v_s = \ddot{a}_s + \dot{\theta}_r a_r$$

(27)

Similarly, solving Equation 22 for $\dot{v}_r$, and substituting into Equation 26 yields

$$\dot{v}_r + \ddot{\theta}_r v_r = \ddot{a}_r - \dot{\theta}_r a_r$$

(28)

Equations 27 and 28 are uncoupled, second order, ordinary, differential equations. The solution to Equation 27 is given by

$$v_s = A \sin \theta_s + B \cos \theta_s t + C \cos at + D \sin at$$

(29)

where

$$C = \frac{\alpha a}{\theta_s - \alpha^2}$$

(30)

and

$$D = \frac{b\dot{\theta}_s}{\theta_s - \alpha^2}$$

(31)

**FIGURE 4** The roll, pitch, and yaw angles.
Two initial conditions on \( v_r \) or its derivatives are required to solve for \( A \) and \( B \) in Equation 29. The first initial condition represents the impact velocity and is given by

\[
(v_r)_{t=0} = E
\]

The second condition represents the pre-impact acceleration and can be obtained from Equation 22 as

\[
(v_r)_{t=0} = (a_r)_{t=0} = 0
\]

Applying Equations 32 and 33 to Equation 29 yields

\[
A = -Da/\theta_i
\]

\[
B = E - C
\]

If

\[
\theta_i = 1.5 \text{ rad/sec} \quad \alpha = 7.854 \text{ rad/sec}
\]

\[
a = -10 \text{ g} \quad E = 88 \text{ ft/sec}
\]

\[
b = -10 \text{ g}
\]

then Equation 29 becomes

\[
v_r = -42.55026 \sin 1.5t + 45.44974 \cos 1.5t + 42.55026 \cos 7.854t + 8.12648 \sin 7.854t
\]

At \( t = 0.4 \text{ sec} \), which represents the half period of the sine wave acceleration pulse, \( v_r = -29.065 \text{ ft/sec} \).

The same problem using Equations 16 through 18 and a fourth-order Runge Kutta solution with a 1-msec time step yields the value \( v_r = -29.083 \text{ ft/sec} \). The numerically calculated value represents an error of slightly more than 0.015 percent in the change in velocity and 0.062 percent in the final velocity. Numerically solving the coupled differential equations of motion is highly accurate for the ranges of values typical in full-scale crash test data reduction.

It is interesting to compare the correct velocity obtained using the exact solution with the value obtained by simply applying Equation 3 as suggested in NCHRP Report 230:

\[
88 + \int_0^{0.4} a_r dt = 88 + 322 \int_0^{0.4} \sin 7.854t = 6.000 \text{ ft/sec}
\]

Obviously, the error induced by neglecting the coupled terms in the equations of motion is very large, at least in this case. The example does illustrate two very important facts:

1. The numerical solution of the coupled differential equations of motion proposed in the preceding paragraphs is very accurate; and
2. It is incorrect to calculate velocity changes by simply integrating the accelerometer data when yaw rates exist.

**Magnitude of Errors in Typical Full-Scale Crash Tests**

The NCHRP Report 230 formulation essentially assumes that the roll, pitch, and yaw rates are negligibly small in comparison to the translational acceleration. This is reasonable for the roll and pitch rates, but it is not true for the yaw rate in typical redirectional tests. When all the angular accelerations are neglected in Equations 19 through 21, the time rate of change of the velocity simply becomes the acceleration.

To assess the relative importance of the coupling terms, assume that the average yaw rate observed in a test is 1.5 rad/sec. Ross et al. (6) have reported much higher yaw rates for very small cars in mini-car collisions with the concrete safety shape; however, 1.5 rad/sec seems to be a reasonable value for the average yaw rate for a wide variety of vehicle types and collision scenarios. In a typical redirectional test, the vertical velocity, \( v_r \), can be assumed to be negligible and the maximum lateral velocity, \( v_r \), is generally less than 20 ft/sec. The second term of Equation 19 is slight since the vertical velocity and pitch rate are very small. The third term is approximately 30 ft/sec or a little less than 1 g. A typical longitudinal deceleration in a redirectional crash test is generally less than 10 g. Thus, the error associated with neglecting the coupled terms in Equation 19 results in a possible instantaneous error of 10 percent in the longitudinal velocity rate of change. The rate of change in the lateral direction is even more sensitive to the coupling terms. If the last term of Equation 20 is neglected and \( v_r \) is assumed to be 88 ft/sec, the second term becomes 132 ft/sec², or more than 4 g. If 10 g is a typical lateral acceleration, the instantaneous error associated with neglecting the coupling term of Equation 20 could be more than 40 percent. Although errors do not accumulate in the instantaneous acceleration, they do in the velocity. If the hypothetical crash event discussed above is 500 msec long, the longitudinal velocity could be in error by 15 ft/sec and the lateral velocity could be in error by 64 ft/sec. Of course, the vehicle has no yaw rate and no lateral velocity prior to the vehicle collision with the appurtenance so the effect of accumulating errors in the change in velocity is unclear.

Two actual redirectional crash tests were selected to compare the effects of including and excluding the coupled terms of the equations of motion on various evaluation parameters. Both tests were redirectional tests corresponding to NCHRP Report 230 test 10 and were chosen because they are fairly typical tests. The errors reported would be greater for collisions exhibiting higher yaw rates such as those involving very small vehicles and rigid barriers (6). For collisions with little or no yaw rate, the error would be very small. For example, the yaw rate in nonredirectional tests such as a head-on collision with an impact attenuator (tests 50 and 51) or breakaway sign support (tests 60 through 63) would result in negligible errors due to little, if any, yaw rotation. The yaw rate is more likely to be significant in redirectional tests where changing the pre-impact direction of the impacting vehicle is the primary objective of the test. Using lighter vehicles and stiffer longitudinal barriers also increase the observed yaw rates. Higher yaw rates and lateral velocities maximize the effect of the coupling terms in Equations 19 through 21.

Test SPI-1 (16) was performed using NCHRP Report 230 test 10 conditions; namely, a 4,500-lb car impacting a standard G4(15) guardrail at 25° and 60 mph. This test was chosen because it is a typical redirectional test using a standard strong post guardrail system. The second test, WE4-1 (17), used a 3,400-lb passenger car ballasted to 4,000 lbs. The vehicle impacted a standard G4(13) guardrail at 25° and 60 mph. This test was performed in a study aimed at investigating alternatives to the vanishing 4,500-lb car and, therefore, the test
conditions do not exactly correspond to test 10 conditions. This particular test was chosen because the vehicle experienced snagging problems during the impact. The frame-horn punched through the barrier causing the vehicle to spin into the traveled way. High yaw rates were observed in this test due to snagging.

As shown in Tables 1 and 2, the errors associated with neglecting the coupled terms in the equations of motion are quite significant when calculating the change in vehicle velocity. Since errors in acceleration accumulate in the velocity terms, the cumulative error increases later in the impact event.

The occupant impact velocity is not as sensitive to this phenomena since the occupant usually collides with the passenger compartment relatively early in the event when the cumulative error is still small. For example, in test SPI-1 the final conditions of the vehicle were calculated 577 msec after impact when the cumulative error was very high, but the occupant lateral impact occurred 170 msec after the vehicle and appurtenance collided. The occupant impact velocities, calculated using the procedure currently shown in NCHRP Report 230 (including the recommended flail distances), were generally reasonable for the first impact. The value of the improved method was most striking for calculating the vehicle exit conditions, changes in velocities, and occupant kinematics after the first occupant/interior collision.

The error associated with nontracking or side impacts could be even greater than those described above. If a nontracking test were conducted with a pre-impact yaw rate of 1.5 rad/sec and lateral velocity of 45 ft/sec (30 mph), the error due to neglecting the coupled terms on the values of instantaneous accelerations would be approximately 20 percent. The cumulative error in change in velocity for such a case would also be large since the vehicle would have a significant yaw rate and lateral velocity throughout the event.

### ORIENTATION EFFECTS

The formulation represented in Equations 1 through 4 does not consider the effect of vehicle rotation on the occupant impact velocity. As shown in Figure 5, the occupant is assumed to travel in the pre-impact direction at the pre-impact speed. The vehicle, however, is rotating and its speed is decreasing. When the vehicle interior and occupant collide, the vehicle lateral and longitudinal directions have changed from those at the beginning of the impact event.

To correctly calculate the hypothetical occupant impact velocity, it is necessary to determine when the vehicle interior and the occupant collide. The following discussion is taken largely from Calcote et al. (2), who formulated an algorithm that includes the effects of the orientation of the vehicle. Transducer data is numerically integrated to obtain the velocity components of the vehicle in the vehicle-fixed reference frame. The yaw rate gyro data is also numerically integrated to obtain the angular displacement of the vehicle. Given the components of velocity and orientation of the vehicle at each time step, the velocities can be transformed from the vehicle-fixed r-s-t reference frame to the global x-y-z reference frame using the following transformation:

\[
\begin{align*}
\mathbf{v}' & = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_x \\
v_y \\
v_z
\end{bmatrix}
\end{align*}
\]

### TABLE 1 COMPARISON OF NCHRP REPORT 230 AND IMPROVED FORMULATION—TEST SPI-1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coupled</th>
<th>Uncoupled</th>
<th>Error (%)</th>
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<tbody>
<tr>
<td>Impact Velocities</td>
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<td></td>
<td></td>
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<tr>
<td>Longitudinal (fps)</td>
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<td>86.2</td>
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<td>Change in Velocity</td>
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<tr>
<td>Longitudinal (fps)</td>
<td>30.9</td>
<td>24.7</td>
<td>20.2</td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>18.7</td>
<td>54.8</td>
<td>193.9</td>
</tr>
<tr>
<td>Occupant Impact Velocities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal (fps)</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>15.3</td>
<td>15.9</td>
<td>3.9</td>
</tr>
</tbody>
</table>

### TABLE 2 COMPARISON OF NCHRP REPORT 230 AND IMPROVED FORMULATION—TEST WE4-1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coupled</th>
<th>Uncoupled</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Velocities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal (fps)</td>
<td>88.0</td>
<td>88.0</td>
<td>-</td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>-1.9</td>
<td>-1.9</td>
<td>-</td>
</tr>
<tr>
<td>Final Velocities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal (fps)</td>
<td>-15.9</td>
<td>-4.7</td>
<td>70.7</td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>-28.5</td>
<td>-52.1</td>
<td>82.9</td>
</tr>
<tr>
<td>Change in Velocity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal (fps)</td>
<td>103.9</td>
<td>92.7</td>
<td>10.8</td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>30.33</td>
<td>5.2</td>
<td>76.3</td>
</tr>
<tr>
<td>Occupant Impact Velocities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal (fps)</td>
<td>27.4</td>
<td>27.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Lateral (fps)</td>
<td>18.8</td>
<td>20.0</td>
<td>6.1</td>
</tr>
</tbody>
</table>

### FIGURE 5 Occupant and vehicle trajectories.
A more general approach is to include all the rotational and translational DOFs in transforming the vehicle velocities into the global coordinate reference frame. The complete transformation between the vehicle-fixed r-s-t coordinate system and the global x-y-z coordinate system is given in matrix form by the following system of equations (10, 11):

\[ \{v_{x,y,z}\} = [T] \{v_{r,s,t}\} \]

\[
\begin{bmatrix}
    v_x \\
    v_y \\
    v_z \\
\end{bmatrix} =
\begin{bmatrix}
    \cos\beta \cos\theta_p & \cos\theta_p & \cos\beta \sin\theta_p \\
    \cos\beta \sin\theta_p & \cos\theta_p & \cos\beta \sin\theta_p \\
    -\sin\theta_p & \sin\theta_p & \cos\beta \\
\end{bmatrix}
\begin{bmatrix}
    v_r \\
    v_s \\
    v_t \\
\end{bmatrix}
\]

where the matrix \([T]\) transforms velocities from the \(r-s-t\) to \(x-y-z\) reference frame and the angles were defined earlier in Figure 4.

If the roll and pitch angles are set equal to 180° and 0°, respectively, the same system of equations shown in Equation 36 is obtained. For most typical crash tests, the earlier derived transformation matrix is sufficient. Equation 37 represents a more general system of equations, however, that can be used in analyzing all types of impact events.

Once velocity components of the center of gravity have been transformed into the global reference frame, an analogous procedure is undertaken to determine the velocity history of the occupant. The occupant is presumed to travel in the pre-impact direction at the pre-impact speed. Calcutt et al. (2) kept track of the relative displacement of the occupant with respect to the vehicle by comparing the position of the vehicle with that of the occupant, both in the global reference frame. When the relative displacement was equal to either 1 ft in the lateral direction or 2 ft in the longitudinal direction, the relative velocity was calculated.

A similar but more general approach is to represent the passenger compartment by an arbitrarily shaped polygon as shown in Figure 6. At the start of data analysis, the nodal coordinates of a polygon are identified with respect to the vehicle-fixed reference frame. The position of each node on the polygon at each time step can be calculated using the following transformation matrix:

\[ \{p_{x,y,z}\} = [T] \{p_{r,s,t}\} \]

\[
\begin{bmatrix}
    \rho_{x,i} \\
    \rho_{y,i} \\
    \rho_{z,i} \\
\end{bmatrix} = [T]
\begin{bmatrix}
    \rho_{x,i} \\
    \rho_{y,i} \\
    \rho_{z,i} \\
\end{bmatrix}
\]

where

- \(\rho_{x,i}\) = position of node \(i\) of the passenger compartment in the \(x-y-z\) frame.
- \(\rho_{r,s,t}\) = position of node \(i\) of the passenger compartment in the \(r-s-t\) frame.
- \(\rho_{r,s,t}\) = position of the vehicle center of gravity, and
- \([T]\) = the transformation matrix defined in Equation 37.

Now the \(x-y-z\) position of each node in the polygon representing the passenger compartment and the \(x-y-z\) position of the occupant are known for each time step. If the occupant is inside the polygon, occupant impact has not yet occurred. When the occupant has just contacted the edge of the polygon, impact with the passenger compartment has occurred. A variety of algorithms can be used to determine if a point is inside, outside, or on the boundary of an arbitrarily shaped polygon (18).

When the occupant contacts the polygonal boundary, the occupant impact velocity is calculated as the occupant’s absolute velocity, normal to the boundary, subtracted from the absolute velocity of the boundary. After this impact has occurred, the occupant is free to translate along the boundary or back into the passenger compartment as shown in Figure 6. Therefore, multiple occupant impacts can occur since the polygon only constrains the occupant from moving beyond the boundary.

The polygonal representation has other more subtle advantages. In the preceding paragraphs, it was assumed that each node on the polygon retained its relative position to the vehicle center of gravity. In general, this need not be so. For side impacts, for example, one node may respond to the accelerations of the appurtenance while the other nodes respond to the accelerations experienced at the vehicle center of gravity. Figure 6(c) represents such a situation; the middle node on the occupant side could represent a luminaire pole intruding into the passenger compartment space. The appropriate occupant impact velocity in this case is the relative velocity of the occupant with the appurtenance rather than the vehicle. The importance of this effect has been noted experimentally in a recently completed series of side impact tests (19). In this test series, Hinch and Stout noted that low occupant impact velocities were often associated with severe, probably fatal, collisions. This algorithmic improvement would allow researchers to obtain more meaningful occupant risk parameters for nontraditional types of impacts.

**IMPROVED FORMULATION**

Several computer programs have been written that address one or more of the issues discussed above. In fact, every agency involved in testing, designing, or researching aspects of roadside appurtenances has a somewhat different method for calculating the vehicle and occupant kinematics and reducing crash test transducer data. This multiplicity and diversity of data reduction tools has not been beneficial to the highway community at large since no agency computes results in exactly the same manner.
A computer program, the Vanderbilt University Crash Test Reduction Program (VUCTRP), was written in the C programming language and incorporates the above improvements (20). The C language was chosen for a number of reasons. First, it is highly transportable, allowing the same source code to be recompiled on a variety of computer hardware. This is an advantage in reducing crash test data since this program could be used to reduce transducer data on, for example, an IBM PC, while it may also be used as a post processor to simulation programs such as Barrier VII, Guard, or HVOSM on a mainframe or minicomputer. Second, C compiles into efficient machine instructions. This is particularly advantageous in such applications as crash test data reduction where a great deal of computation is required. C's structure and syntax make it ideal for creating modular systems that can be easily modified and updated by adding new modules of source codes. For example, if an agency does not use the transducer arrangement assumed by the appropriate function in the current program, the agency could add its own function to process data with a different transducer arrangement.

Table 3 shows the sequence in which the output files are created and the basic information in each file, and Figure 7 shows the basic flow of data through the program. The user supplies the name of the file containing the impact conditions and other initial values and the name of the file containing the transducer data. All the functions presented in Figure 7 operate independently; each file receives its input from a temporary file, performs its calculations, and then writes its results to another temporary file.

The purpose for this structure is to promote improvements. If a testing agency does not, for example, use two accelerometers and one rate gyro, it would be a straightforward operation to write a function to replace the default input function. The modules of the program prior to the change and after would not need to be modified as each module is independent of the others.

The first function in the program is called one_pac(). This function reads in the digitized and filtered transducer data, reduces it to represent the accelerations at the center of gravity, and solves the coupled equations of motion. The results are written to an intermediate file. The one_pac() function assumes that longitudinal and lateral transducers and a yaw rate gyro were used, so the application of Equations 19 through 21 follows directly. If a testing agency uses a different configuration, another set of coupled differential equations would be required and another function written. There are currently two transducer arrangement options that can be selected when running the program. These arrangements are assumed in the following functions:

1. Function one_pac()—longitudinal and lateral accelerometers and a yaw rate gyro,
2. Function two_pac()—two sets of three linear accelerometers positioned at two points on the vehicle,
3. Function six_pac()—three linear accelerometers and three rate gyro's, and
4. Function no_pac()—no instrumentation.

The purpose of the last function, no_pac(), is to supply the program with the vehicle kinematics from another source such as a simulation program. Only the functions one_pac() and no_pac() have been written at this time. Using this style

<table>
<thead>
<tr>
<th>File name</th>
<th>Description of Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>filename.in</td>
<td>A file containing the accelerations and rotation rates measured during the crash test. The file is generated by the user from transducer test data.</td>
</tr>
<tr>
<td>filename.bc</td>
<td>File containing the boundary conditions: the vehicle's six initial positions, six initial velocities, and basic time step. This file is also generated by the user.</td>
</tr>
<tr>
<td>filename.rst</td>
<td>A file of the accelerations and velocities of the vehicle center of gravity. The file contains three accelerations and six velocity terms for each time step where data was obtained. This file is generated by either the function one_pac(), two_pac(), no_pac(), or six_pac(). The user chooses the appropriate function on the command line.</td>
</tr>
<tr>
<td>filename.xyz</td>
<td>A file of the six components of the vehicle velocity and the six components of the vehicle position in the global coordinate system. Each line represents the state of the vehicle at a particular time step. The file is generated by the function rst_to_xyz().</td>
</tr>
<tr>
<td>filename.vks</td>
<td>A file which summarizes the vehicle kinematics. The file contains the accelerations and velocities of the vehicle center of gravity, the vehicle position in the global coordinate system. The 50-msec average accelerations are also appended to this file. The file is generated in the function vehicle_kinematics_summary().</td>
</tr>
<tr>
<td>filename.pcp</td>
<td>A file containing the global positions of each node on the occupant compartment polygon at each time step. The file is generated in the function compartment_position().</td>
</tr>
<tr>
<td>filename.op</td>
<td>A file containing the global position of the occupant at each instant of time. The file is generated in the function check_interface().</td>
</tr>
<tr>
<td>filename.ors</td>
<td>A file containing a summary of the occupant risk values. The file contains the times of occupant contact with the vehicle interior as well as the relative velocities at those times. The file is generated in the function check_interface(). The maximum ride-down accelerations are appended to this file in the function average_accel().</td>
</tr>
</tbody>
</table>
allows the user and the researcher to easily customize the program without having to recode large segments of the program.

Regardless of the particular function, a file is written that contains the three translational accelerations, three translational velocities, and three angular velocities. Thus, the particular method used to convert the transducer data into velocities of the center of gravity has no effect on the remainder of the program.

The next step is to transform velocities in the vehicle-fixed reference frame into global velocities. This is accomplished in the function `rst_to_xyz()`. This function also calculates the translational and angular displacements of the vehicle and writes them to a file. Again, all six displacements (three translational and three angular) are written to the intermediate file so that later modules of the program are independent of earlier modules.

Once the displacements and velocities are known in the global reference frame, a summary report of the vehicle kinetics is assembled using the function `vehicle_kinetics_summary()`. The function `average_acceleration()` computes the 50-msec average and appends it to the end of the vehicle summary report.

Up to this point, the program is simply processing vehicle data; nothing has been done to calculate occupant risk values. This portion of the program begins in the function `compartment_position()`, where the position of the nodes representing the corners of the passenger compartment are calculated in global coordinates for each time step. The global position of each node at each time step is written to a file.

The function `check_interface()` first calculates the occupant global position in a particular time step. Once this position is defined, the global positions of the passenger compartment nodes are read from the files stored in the previous step. A
check is performed to see if the occupant position is inside, outside, or on the boundary of the polygon defined by the passenger compartment nodes. If the occupant is inside the polygon, occupant impact has not yet occurred and the program continues to the next time step. If the occupant position is outside of or on the polygon boundary, occupant impact has occurred.

When impact has occurred, the velocity normal to the boundary is calculated and subtracted from the occupant velocity in that direction—the occupant impact velocity. When this value has been found and stored, the occupant position is set on the boundary, and the next time step is then analyzed. The occupant is allowed to translate in any direction in the next step, but if the occupant position again falls outside the boundary, the position is set to be on the boundary and the occupant impact velocity is again calculated. All the impact velocities and the times of their occurrence are saved in a file with the global position of the occupant at each time step. This procedure is shown in Figure 8.

The occupant summary report is assembled in the function occupant_summary() by reading values from the various intermediate files. The ridedown accelerations are calculated in the function average_accelerations() and appended to the summary report. After the occupant summary report has been assembled and printed, the system deletes the intermediate files and exits to the operating system.

CONCLUSIONS

This paper has presented a discussion of several improvements to the way that crash test data is analyzed and occupant risk parameters calculated. In most cases, the improvements result in only small changes to the occupant risk values previously obtained when analyzing tests recommended in NCHRP Report 230. This is because the current procedure only recommends calculating occupant risk parameters for the first occupant/interior collision, which generally occurs early in the impact event when the cumulative error is still small. The primary advantage of using the formulation presented in this paper is that the improvements make the model more physically correct, versatile, and general. In other cases, neglecting certain phenomena can result in large errors. The current technique is severely limited by the assumptions imposed on it: the improved formulation is more flexible and should prove to be a useful tool in a wide variety of collision scenarios that cannot currently be addressed. For example, the procedure recommended in NCHRP Report 230 should not be used to investigate nontracking and side impacts because the method does not accurately account for the vehicle yaw rate. More versatile and general models will be important when new types of impact conditions are added to the standard test matrix.

In summary, this paper has illustrated the importance of the following physical effects on analyzing crash test data:

- Mislocating transducers on the test vehicle by 12 in can cause a 10 percent error in the acceleration values measured. Care should be taken by testing agencies to accurately measure and position these transducers. If the locations of the center of gravity and transducer are documented, Equations 10 and 11 can be used to find the response of the center of gravity.
- The quality of the results provided by any data reduction program are a function of the original transducer data. When rate gyros are used in full-scale crash tests, the data should be smoothed before using an analysis technique such as the one presented in this paper. Smoothing is required to estimate the derivative of the angular velocity time curve. Unsmoothed rate gyro data can introduce errors into the accelerometer data since information regarding the derivative of angular velocity, angular acceleration, is lost. The improvements described herein are sensitive to this effect where other techniques are not.
- Ignoring the coupled terms in the equations of motion can result in errors in the instantaneous accelerations, and the cumulative effect of these errors on the velocity history of the test vehicle can be exceptionally large. When reducing vehicle acceleration data, the coupled equations of motion should always be used.
- The effect of neglecting more sophisticated techniques for tracking the occupant and vehicle interior positions result in only small changes in values for the first occupant/interior collision for the current NCHRP Report 230 tests. The policy issues of what the appropriate flail space might be, how long occupant kinematics should be tracked, what the occupant initial position should be, and what the vehicle compartment geometry should look like have not been dealt with in this paper. The advantage of the technique proposed is that these policy questions can be explored using this improved method and they cannot be explored with the current formulation of the flail space method.

The computer program written to implement these improvements is proposed as a means for standardizing data reduction activities throughout the roadside appurtenance research community.

REFERENCES

4. M. H. Ray, J. D. Michie, and M. W. Hargrave. Events that Produce Occupant Injury in Longitudinal Barrier Accidents. In
The authors are to be commended for their efforts to enhance the flail space model. Incorporation of refinements into the mathematics and will ignore the limiting fundamental premise of the model.

As presented in earlier papers, the flail space concept is a simplistic model of a most complex event. Basically, the occupant is assumed to be a simple, unrestrained lumped mass, with undefined properties, that is prepositioned in the passenger compartment and moves toward and strikes a vertical windshield/dash, side door surface, or both because of vehicle collision accelerations. The mass/vertical surface impact velocities are further assumed to be direct indicators of the injury-producing mechanism: the higher the impact velocity, the more severe the injury. As a second part of the concept, applicable in cases where the vehicle undergoes large speed changes (i.e., during a frontal crash cushion impact), it is assumed that the mass (and occupant) remain in contact with the initial impact surface and then directly experience the vehicle acceleration intensities during the final stopping of the vehicle.

The flail space model was only intended to grossly quantify the initial impact of a front seat occupant during a vehicle collision with a roadside feature. The underlying concept breaks down during subsequent impact collisions because the point mass kinematics are not representative of the multi-degree-of-freedom occupant, and current injury assessment cannot handle cumulative damage. Other factors not addressed by the flail space model include size and physical condition of the occupant and configuration of the compartment (i.e., knee bolsters, bucket seats, windshield rake, seat design, etc).

In summary, I believe that the flail space model is a valid tool for assessing occupant severity during vehicle to roadside feature collisions. However, researchers and other potential users who may wish to extend its range of application should be well aware of its basic premises and limitations and use caution.

AUTHORS’ CLOSURE

There are two distinct issues addressed in the foregoing paper:

1. Correctly calculating the response of a vehicle to accelerations measured during a full-scale crash test, and
2. Generalizing the flail space method.

Most of the mathematics presented in the paper deal with the first of these issues. It is impossible to correctly determine the response of the occupant if the response of the vehicle is not correctly determined. The procedure currently used works for only a few special cases. The improvements suggested would allow researchers and developers to correctly calculate vehicle kinematics and hence occupant kinematics for all types of impact scenarios.

Michie’s comments relate primarily to the second issue; namely, the generalization of the flail space method. The connection between occupant injuries and the occupant-interior impact velocity is only tenuously understood. The improvements suggested in the foregoing paper have not changed any of the fundamental assumptions of the flail space method originally proposed by Michie nearly 10 years ago; the occupant is still presumed to be an unrestrained lumped mass subjected to accelerations transmitted to the vehicle by an appurtenance collision. The improvements indicated have made the calculation of the occupant risk parameters more realistic and feasible for impacts in general. The great utility of the flail space method arises from the fact that it provides one quantitative description of the potential for serious occupant injury and not because it faithfully represents the actual motion of the occupant.

One of the bases of the flail space method preserves that the potential for serious occupant injury is related to the