Paper Number 971100

Validation of a FEA Tire Model for Vehicle Dynamic Analysis and Full Vehicle Real Time Proving Ground Simulations

Chang-Ro Lee and Jeong-Won Kim
(Ssang Yong Motor Company)

John O. Hallquist
(Livermore Software Technology Corporation)

Yuan Zhang and Akbar D. Farahani
(Engineering Technology Associates, Inc.)

ABSTRACT

A tire model and its interface performance with road surface plays a major role in vehicle dynamics analysis and full vehicle real time proving ground simulations.

The successful tire model must be able to support the vehicle weight, provide vehicle control and stability, transfer various forces and torques from road/tire interaction to a vehicle chassis/ suspension system. The dynamic effects in terms of tire stiffness and internal damping characteristics in impact loading conditions must also be accounted for in the model.

A Finite Element Analysis (FEA) tire model is established and its performance is validated using LS/DYNA3D* analysis code simulating the radial and lateral static stiffness test conditions, the one-meter dynamic free-drop test condition and the rolling cornering stiffness. The analysis results are compared with available test data and a generic empirical formula.

INTRODUCTION

The tire-wheel system is one of the most important subsystems of a ground vehicle. Different control, drive and resistance forces created from the tire-ground interaction are carried and transferred to the vehicle by tire. A modern tire-wheel system plays a key role in vehicle load carrying ability, handling and steering stability, drivability and comfort. Viewing the tire-wheel system as a load carrying and transfer device, it provides the following three basic functions [1]** :

1. support and transfer vertical loads, absorb and reduce ground impact and the consequent vehicle vibration
2. provide longitudinal forces for acceleration and braking
3. provide lateral forces for cornering and steering

The complexity of modern pneumatic tires has limited the application of analytical methods. Over the past ten years or so, the Finite Element Analysis (FEA) method has been integrated into the tire design/testing process with increasing pace [2]. Finite element tire model analysis has been used to decrease the tire development cycle and even to replace certain tire tests. Noor and Tanner [3] have reviewed FEA tire modeling used by other researchers. While many different FEA tire models have been developed to study the general tire behavior such as stress and deformation due to inflation, natural frequencies, footprint shape and rolling contact models [4-8], rolling resistance [9, 10], tire/rim interaction [11, 12], or even to simulate some destructive tire testing [13], very few tire models were developed to be directly used in vehicle dynamics analysis [14] and none, to the authors knowledge, were used in real time Proving Ground simulation.

Virtual Proving Ground*** (VPG) is an analysis methodology developed at Engineering Technology Associates, Inc. for full vehicle real time proving ground simulations. VPG applications require a totally different FEA tire model than those used in tire design/testing studies. To investigate general tire behavior or even to study tire dynamics [14], it was common and usually sufficient to model a small portion of the tire in fine finite element mesh and leave a very coarse

* LS/DYNA3D is a trademark of Livermore Software Technology Corporation

** Numbers in brackets refer to references at the end of this paper

*** Virtual Proving Ground (VPG) is a trademark of Engineering Technology Associates, Inc.

mesh for the rest part of the tire model. In many cases, only a two-dimensional model of the tire cross-sectional profile is necessary to fulfill the analysis [14]. Also in most of the previous studies, no wheel model was used. For a real time VPG analysis, the tire model has to be a three-dimensional axisymmetric one because it will roll on the ground in full cycles. The wheel also has to be modeled together with the tire model in order to attach this subsystem to a chassis/suspension FEA model or, as in most of the VPG applications, to a full vehicle FEA model.

The tire model used in vehicle dynamics analysis and real time VPG applications is also differentiated from the design/testing oriented tire models in several other aspects. For example, most of the previously developed tire models each had a portion with very fine finite element mesh. The tire model would have had tens of thousands of shell/solid elements if this fine element mesh had been extended axisymmetrically to the entire model [15]. It would need much larger and faster computers and much more CPU power to use four such tire models, plus the entire full vehicle model, in a real time analysis.

Another difference between the two types of tire models is that while it is necessary for a design/testing FEA tire model to focus on the detailed material properties of every component for a particular type of tire, it is not necessary for a VPG tire model to use the exact material properties for tires from different manufacturers and types. The characteristics of a vehicle dynamics or VPG tire model is determined by the fact that the analysis is focused on the vehicle instead of the tire itself. In these applications, tire functions mainly as a device which carries the weight of the vehicle and transfers the various loads produced during tire-ground interaction to the vehicle chassis/suspension system. Keeping this in mind, a simple, efficient and yet accurate enough tire model is then possible and, maybe, more practical.

In the following sections, a FEA tire model for the vehicle dynamics analysis and VPG application purposes was developed. The tire model was also validated through several simulations using LS/DYNA3D. These validations were based on the available test data as well as the main features a VPG tire should have. As discussed above, the interested characteristics of a VPG tire model are some of the global properties. The main features included in this paper are the static radial (vertical) stiffness, the static lateral stiffness, the dynamic response in a free drop condition, and the (steady rolling) cornering stiffness. Correlation of the analysis results with the available test data and a generic empirical formula are also given. ETA/FEMB* were utilized to pre- / post-process all the simulation results.

FINITE ELEMENT MODEL

The construction of the FEA tire model includes two steps: preparation of the input data file and generation of the FEA model.

The input data file provides the following information:

1) TIRE GEOMETRY

The tire-wheel cross-sectional profile was defined in the global y-z plane where the y-axis is in the tire width direction and the z-axis is in the radial direction. The tire center is defined as the origin. Two kinds of tire cross-sectional profiles are used and are shown in Figure 1. The positions of nodes in the profiles are created by ETA/FEMB and then the coordinates of these nodes are output as a NASTRAN file and read into the input file. Also included in the input geometry data are the thickness of the plies. Asymmetric cross-sectional profiles can also be used.

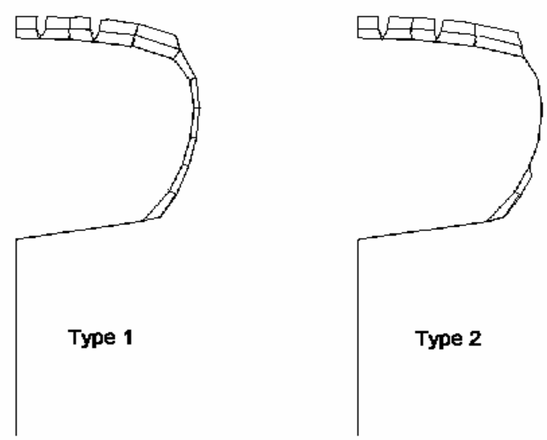


Fig. 1 Models for tire cross-sectional.

2) MATERIAL PROPERTIES

The plies, treads, sidewall, and chafer were put into different parts and material properties were assigned separately for each part to simulate the inhomogeneities in tire materials. For plies, either isotropic or anisotropic elastic materials can be used. Layered composite plies can also be used if detailed material properties are provided. Elastic plies are used presently because they provide the same global characteristics we are currently interested in and save computing CPU time.

The explicit nonlinear three-dimensional dynamic analysis code LS/DYNA3D has been used for the entire study. Rubber materials (chafer, lower and upper treads) are modeled by solid elements and are of Mooney-Rivlin type [16]. Shell/plate elements are used to model the plies and the wheel and are of Belytschko-Lin-Tsay type [17-19].

3) TIRE PRESSURE

Tire inflation pressure can be set to be equal to the exact value (in MPa), as required by user. Tire pressure will change during the course, when the vehicle is running on the road. The pressure change is mainly due to the tire operating temperature variation and to the tire deformation (air volume change). For an ideal gas like air, it is well-known that:

$$pV / T = const. \tag{1}$$

In vehicle dynamics analysis and VPG applications, the assumption is that temperature change is small for a short time period and then ignored. Equation (1) is then used with constant temperature.

* ETA/FEMB is a trademark of Engineering Technology Associates, Inc.

The tire model generation proceeds by utilizing a FORTRAN program developed at Engineering Technology Associates, Inc., to rotate the tire-wheel cross-sectional profile about the y-axis and generate a complete axisymmetric mesh. The plate elements in the input profile are extruded into solid elements and the bar elements into shell elements. The holes in the wheel disk are also axisymmetric and could be of any reasonable number.

In general the tire-wheel model consists about 2100 to 2500 elements (depending on which type of cross-sectional profile being selected), among which about half are solid elements and the other half are shell/plate elements. There are 960 shell/plate elements onto which inflation pressure is assigned.

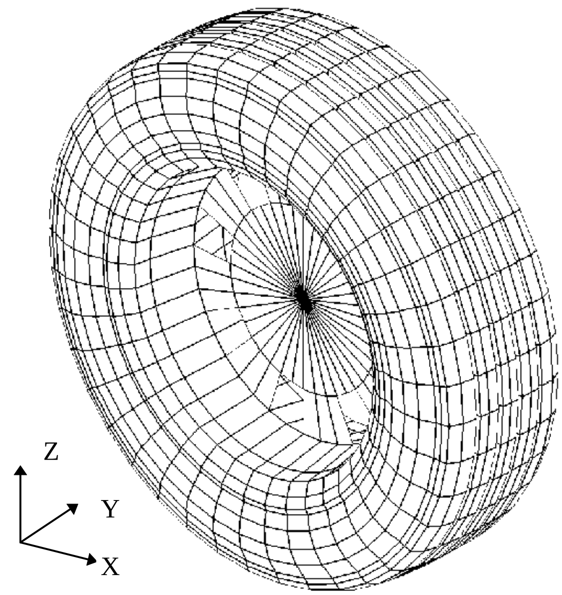
Shown in Figure 2 are the different views of a P215/60 R16 low-profile FEA tire model used in this study. This model utilizes tire profile type 1 in Figure 1. It consists of 1280 solid elements and 2500 shell/plate elements. Figure 3 shows the FEA model of a P195/70 R14 tire which has also been used in the present study. This model consists of 960 solid elements and 2150 shell/plate elements, and is constructed from tire profile type 2 in Figure 1.

Elements of the wheel center part are defined as rigid plate elements. This is necessary when the tire model is mounted to the vehicle drive axle via a revolute or cylindrical joint in the LS/DYNA3D code for a vehicle dynamics analysis and VPG simulation.

In all the validation test simulations, a flat test ground surface modeled as rigid plate elements was used. Utilizing LS/DYNA3D, a contact interface is defined between this surface and the outer surface of the tire upper tread.

QUASI-STATIC RADIAL STIFFNESS ANALYSIS

The overall radial (vertical) stiffness is one of most important tire properties which characterizes the load carrying capacity and load transferring ability of tires. A proper radial stiffness of the FEA tire model is necessary for correct, accurate vehicle dynamics or VPG simulations. The stiffness test information can be readily obtained from tire manufacturers for any type of tire.



The quasi-static analysis described in this section is designed to validate the overall radial (vertical) stiffness of the present FEA tire model. In the present simulation, instead of gradually increasing the load at the wheel center, we equivalently constrain the rigid wheel center part while move the test ground vertically up at a low, constant speed. The load-time history is recorded and translated into a load-deformation curve and the stiffness calculated.

LS/DYNA3D system dynamic relaxation is used to minimize the dynamic effect during the simulation process. Since the kinetic energy calculated during the entire loading period is less than one percent of the total energy, this simulation is considered a quasi-static one and so does the resulting stiffness.

For the low profile tire P215/60 R16, as shown in Figure 2, the designed (vertical) static load is about 5400 N

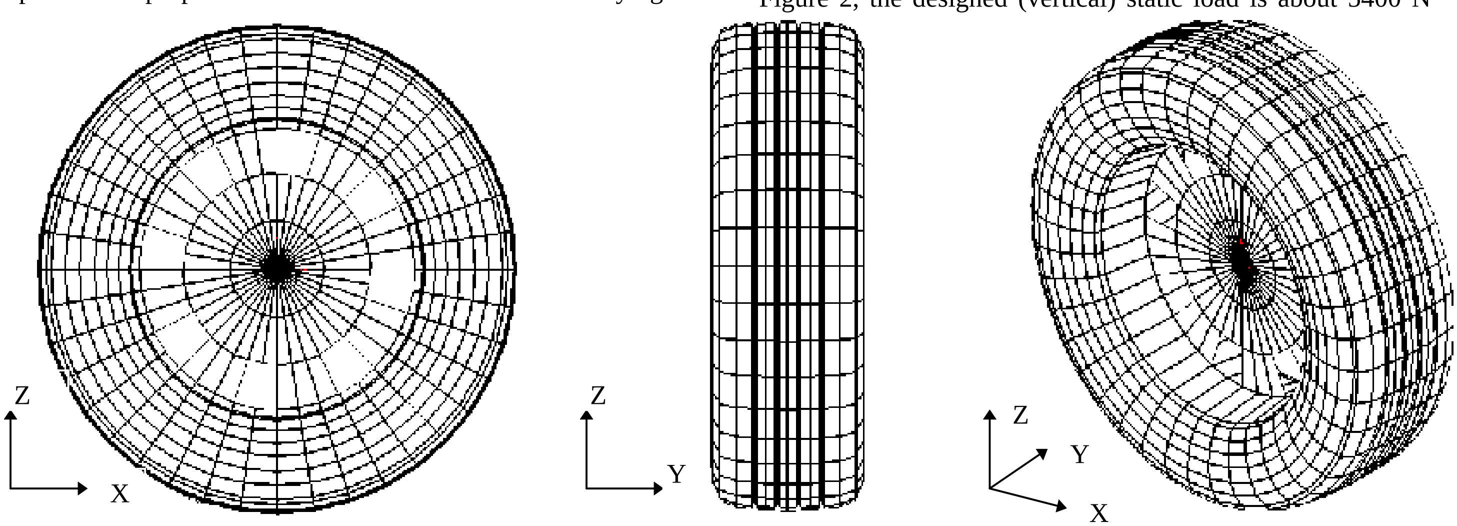


Fig. 2 FEA model for P215/60 R16.

(about 1/4 of the vehicle, passengers and luggage weight). The simulation is carried out well beyond this limit. The calculated load-deflection curve is shown in Figure 4.

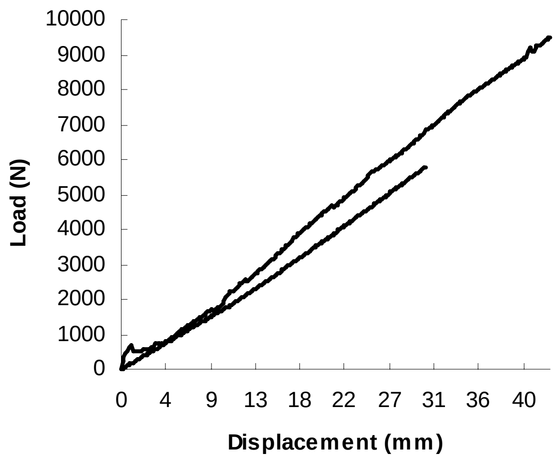


Fig. 4 FEA simulation and test results: Vertical load-displacement curves for P215/60 R16 tire.

It can be seen that linear relationship between the vertical load and the vertical displacement is well kept during the entire simulation load range after initial stabilization is achieved. Also shown in Figure 4 is the test data obtained at Ssang Yong Motor Company, closely matching the analysis results. Listed in Table 1 is the comparison of both test and simulation results for radial stiffness. Both deformed and original tire cross-section profiles are plotted in Figure 5.

Table 1: Results for Tire P215/60 R16 Quasi-Static Radial Stiffness

	Test	Analysis	
		A[1]	B[2]
Radial Stiffness (N/mm)	220	229.4	233.8
Relative Discrepancy		4.27%	6.27%

1 average over the local stiffness where the load is over 4000 N
2 average over the entire loading range

The influence of tire inflation pressure upon tire radial stiffness is shown in Figure 6 for the P195/70 R14 FEA tire model shown previously in Figure 3. One can see from this plot that tire radial stiffness increases almost linearly with

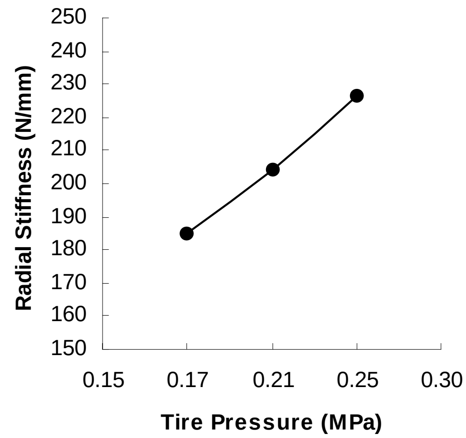


Fig. 6 Analysis results of the effects of inflation on the radial stiffness of the P195/70 R14 tire model.

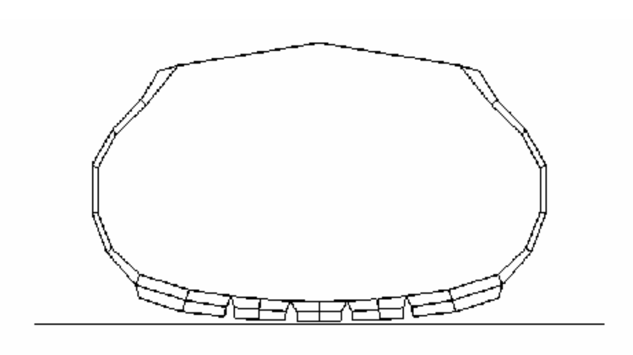
the increase of tire inflation pressure, at least within the simulated pressure range.

Since LS/DYNA3D is a dynamic analysis code, one might also be interested in seeing the influence of loading speed on the quasi-static analysis result. In the above mentioned simulations, a loading speed of 750 mm/s is used. Listed in Table 2 are the radial stiffness simulation results for the P195/70 R14 tire model at different loading speed. The maximum difference is seen to be less than 0.5 percent. This gives us the conclusion that, at least in the present case, loading speed in the above shown range has no influence upon the LS/DYNA3D quasi-static simulation results. Kinetic energy is always less than one percent of the total energy in each of the above simulations. Dynamic relaxation used in the analysis is responsible for the reduction of dynamic effects, although system damping energy is less than two percent of the total energy during the analysis.

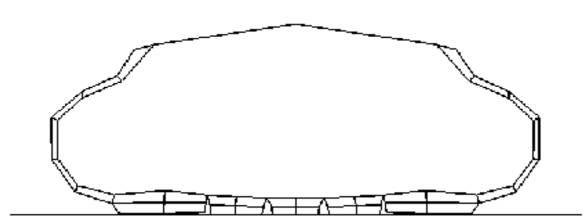
Shown in Figure 7 is the deformed P195/70 R14 tire model during radial stiffness simulation.

QUASI-STATIC LATERAL STIFFNESS ANALYSIS

The quasi-static lateral stiffness of a tire is closely related to vehicle cornering and steering ability. In quasi-static lateral stiffness analysis conducted here, a certain constant vertical load is first applied to the wheel center and a lateral load is then applied gradually at a constant speed. The lateral load-displacement curve is one of the simulation



Original



Deformed

Fig. 5 Original and deformed cross-sectional profiles for P215/60 R16 tire model in radial stiffness simulation.

results. Tire lateral stiffness is then calculated from this curve.

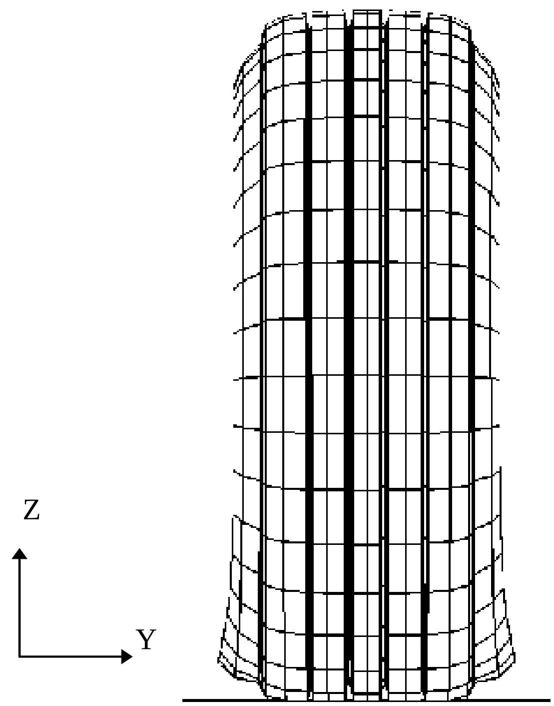


Fig. 7 Deformation of P195/70 R14 tire model in radial stiffness simulation.

Table 2: Simulated Radial Stiffness of P195/70 R14 Tire Model at Different Loading Speed		
	Loading Speed (mm/s)	Stiffness (N/mm)
Simulation # 1	750.000	226.33
Simulation # 2	375.000	225.69
Simulation # 3	187.500	227.15
Simulation # 4	93.750	226.86
Simulation # 5	46.875	225.83
Average		226.37

Shown in Figure 8 is the deformed shape of the P195/70 R14 tire model during lateral stiffness simulation. The wheel disk center part is defined as a rigid body and kept vertical (no rotation about x-axis) during the simulation process.

Tire lateral stiffness is a function of the applied steady vertical load. It is common knowledge that the static lateral stiffness of a tire will be of maximum value at a vertical load near its desired load capacity. Shown in Table 3 are the analyses results of the quasi-static lateral stiffness of the P195/70 R14 tire model at different static vertical loading. It can be seen that this tire model does have a maximum stiffness at a vertical loading near 7500 N.

Also listed in Table 3 are the lateral to radial stiffness (266.33 N/mm for this model) ratios. Experience indicates that the static lateral stiffness of a tire is around 40% of its

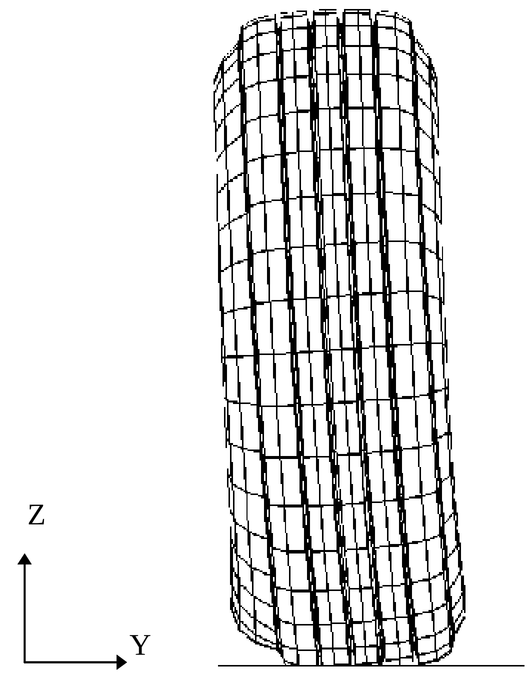


Fig. 8 Deformation of P195/70 R14 tire model in lateral stiffness simulation.

static radial stiffness value [20]. Based on this experience, the validation of the present tire model as shown in Table 3 can be assumed to be excellent.

Table 3: Tire P195/70 R14 Model Quasi-Static Lateral Stiffness Simulation Results		
Wheel Center Loading (N)	Lateral Stiffness (N/mm)	Lateral/Radial Stiffness Ratio
3078	82.76	36.57%
5363	90.36	39.32%
7578	91.74	40.53%
10051	89.43	39.51%

ONE-METER FREE-DROP SIMULATION

The free-drop test is one of the standard dynamic tests of tires. The one meter free-drop simulation conducted in this study is designed to validate the dynamic behavior and properties of the FEA tire model. From the equation of motion in dynamics, one can readily obtain:

$$v = \sqrt{2gh} \tag{2}$$

where h is the initial height, g is the gravitational acceleration, and v is the velocity of the falling object before it touches the ground. According to this equation, an initial velocity of 4427 mm/s is assigned to the tire model and the analysis begins when the tire is 0.1 mm above the ground. The simulation result of the wheel center displacement of the P215/60 R16 tire model is presented in Table 4. Excellent correlation is found between simulation and test results.

Table 4: P215/60 R16 Tire One-Meter Free-Drop Test and Simulation Results	
Test Result (Ssang Yong Motor Company)	750 mm
Analysis Result (tire pressure = 0.25 MPa =36.3 psi)	757.13 mm
Relative Discrepancy	0.95%

Figures 9 and 10 show the wheel center displacement and velocity as functions of time during the simulation, respectively. The velocity of the wheel center oscillates periodically due to the free vibration modes of the tire-wheel system, after impact with the ground. From equation (2), the vertical velocity of the whole tire-wheel system as one rigid body just after impact should be 3834 mm/s in order to bounce back to a height of 750 mm. It is difficult to determine in closed form the exact wheel center after impact speed for a deformable tire-wheel system. From this study, the filtered speed is about 3850 mm/s. This could be the reason that the simulation was continued until the first bounce back height was observed.

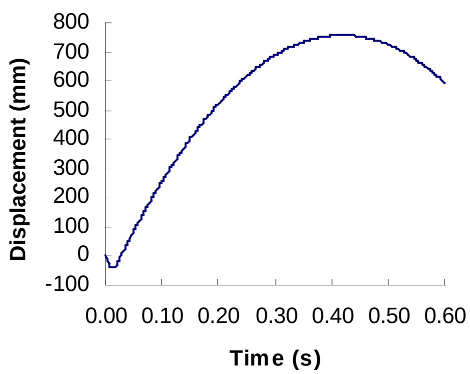


Fig. 9 Vertical displacement of P215/60 R16 tire model in one-meter free-drop simulation.

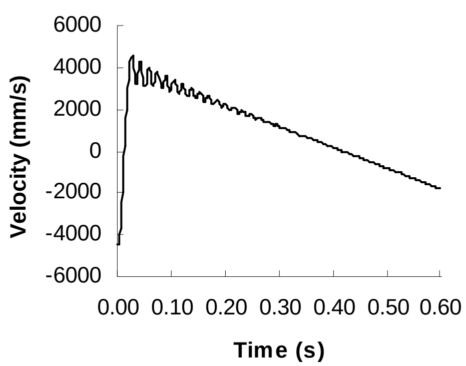


Fig.10 Vertical velocity of P215/60 R16 tire model in one-meter free-drop simulation.

Unlike the static stiffness simulations, no system damping and dynamic relaxation are introduced into the free-drop analysis. Energy is dissipated purely through the inherent hysteresis and internal damping of the rubber materials. The change of tire tread-ground interface properties (for example, the interface friction) does not alter the amount of energy dissipated. The bounce back height is determined by the dynamic characteristics of, as well as the materials used in, the tire model.

The good agreement between analysis and test results indicates that the present tire model does have some proper dynamic characteristics in simulating a real tire.

CORNERING STIFFNESS SIMULATION

In a cornering maneuver, a side force will be developed at the contact patch (footprint) of a rolling tire under lateral force. The tire will move along a direction other than the forward direction in the wheel plane with no relative slip between tire tread and ground. This is the side slip phenomenon during vehicle cornering. The angle between the tire velocity vector and the wheel plane is called the **slip angle** and the side force developed at the tire-ground contact patch with zero wheel camber is call the **cornering force**. The relationship between the cornering force and the slip angle is of fundamental importance to the load carrying and transferring capacity of a rolling tire, and the handling and steering stability of a vehicle. The cornering stiffness of a tire is defined as [22]:

$$k_{cornering} = \frac{\partial F_{cornering}}{\partial \alpha} \bigg|_{\alpha=0} \qquad (3)$$

where α is the slip angle. The unit of the cornering stiffness is conventionally chosen to be N/rad.

Previous FEA studies of tires under cornering forces were focused on the transmission of loads from the tire footprint to the tire bead area and the wheel rim, and thus static analysis was used under certain assumptions [21, 11]. With LS/DYNA3D, we are able to simulate the side slip phenomenon of a steadily rolling tire and calculate its cornering stiffness through nonlinear, transient dynamic analysis.

In the simulation, a constant vertical load is first applied to the wheel center and then the tire is set in rotation at a constant angular velocity, driving the tire in the x-direction at a constant speed of about 6 mph. A lateral load is applied to the wheel center in the y-direction, when the tire is in steady rotation. The slip angle is calculated from the wheel center path in the x-y plane (ground). The cornering force is also recorded from the contact interface force (the y-component). Varying the applied lateral load, we are able to obtain the relationship between the cornering force and the slip angle and to calculate the cornering stiffness from equation (3).

Shown in Figure 11 is the relationship between the cornering force and the slip angle under different vertical loads for our P195/70 R14 tire model. These curves are similar to those given by Wong [22].

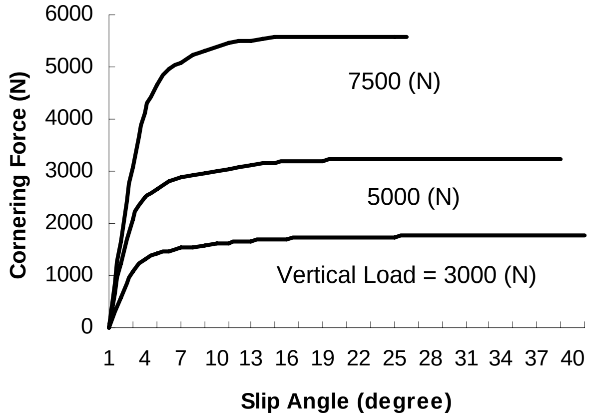


Fig. 11 Cornering force vs. slip angle as a function of vertical loading: Simulation results of P195/70 R14 tire model.

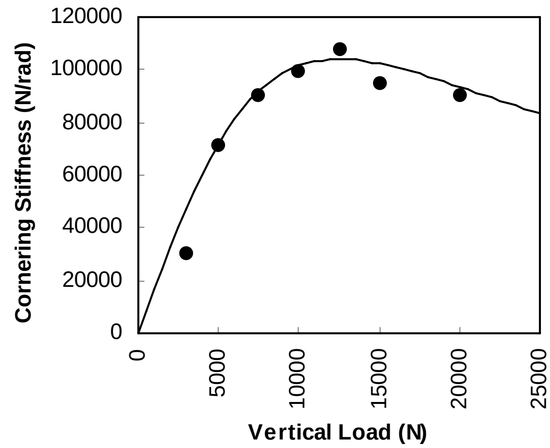


Fig. 12 Cornering stiffness vs. vertical loading. Solid line-"Magic Formula"; Dots-simulation results.

As can seen from Figure 11, the cornering stiffness of a tire varies with vertical loading. An empirical tire model was developed by the Delft University of Technology and Volvo Car Corporation, which can accurately describe the relationship between cornering stiffness and vertical load. Their empirical formula, so called in the literature as the "magic formula tyre model" states that, for zero camber [23]:

$$k_{cornering} = A\sin[2\arctan(\frac{F_{vertical}}{B})] \qquad (4)$$

where A and B are constants which need to be subtracted from test data. The physical meaning of these constants is that when vertical loading equals to B , the cornering stiffness is maximum and equals to A. Since it is impossible to obtain the exact A and B from simulation results of just several vertical loads, the constants are set to be A =10400 (N/rad) and B=12500 (N), which are the maximum cornering stiffness and correspondence vertical load in the simulations. Both the "magic formula" and the simulation results of the P195/70 R14 tire model are plotted in Figure 12. Good correlation is found.

Shown in Figure 13 is the tire-ground contact patch in the cornering stiffness simulation. The asymmetric footprint reveals clearly the side slip phenomenon of the rolling tire under lateral force. During the simulation loading period, no relative slip is observed at the tire tread-ground contact patch.

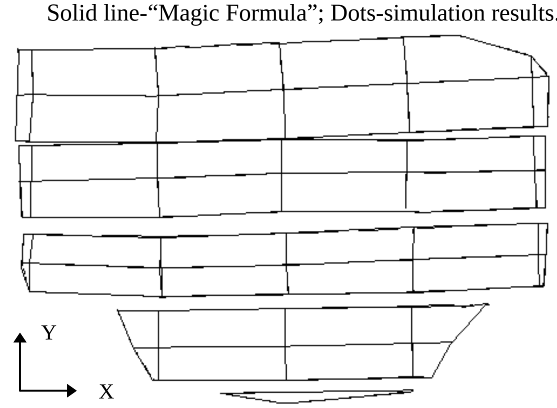


Fig. 13 Tire-ground contact patch (footprint) of P195/70 R14 tire model in rolling tire cornering stiffness simulation.

CONCLUSION

A FEA tire model is developed. This tire model is targeted for the utilization in vehicle dynamics analysis and full vehicle real time Virtual Proving Ground simulation. It contains less structural details of the tire than most other tire design/test models in the literature, and thus consists of much less elements and degrees-of-freedom. But the most important load carrying and load transferring properties of a tire are retained, as shown though our validations.

The present FEA tire model validation is carried out with the LS/DYNA3D nonlinear quasi-static and transient dynamic analysis capability, simulating several tire test procedures. Instead of exploring details like bead/rim interaction, etc., these simulations concentrate on establishing some of the most important static and dynamic features of a tire-wheel system as a whole. These overall static and dynamic properties will play the most important part in vehicle dynamics analysis and VPG application simulations. Good correlation has been established between the simulation results and the available tire test data, as well as an empirical tire formula.

REFERENCES

1. Gillespie, T.D., *Fundamentals of Vehicle Dynamics*, SAE, 1992, Chapter 10, pp.335-376.

2. Kenny, T.M. and Stechshulte, R.A., "Applications of finite element analysis in tire design," *Tire Science and Technology*, TSTCA, Vol. 16, 1988, pp. 96.
3. Noor, A.K. and Tanner, J.A., "Advances and trends in the development of computational models for tires," *Comp. and Struct.*, Vol. 20, 1985, pp. 517- 533.
4. Gall. R., Tabaddor, F., Robbins, D., Majors, P., Sheperd, W. And Johnson, S., "Some notes on the finite element analysis of tires," *Tire Science and Technology*, TSTCA, Vol. 23, 1995, pp. 175-188.
5. Gall. R., Tkacik, P. And Andrews, M., "On the incorporation of frictional effects in the tire/ground contact area," *Tire Science and Technology*, TSTCA, Vol. 21, 1993, pp. 2-22.
6. Faria, L.O., Oden, J.T., Yavari, B., Tworzydlo, W., Bass, J.M. and Becker, E.B., "Tire modeling by finite elements," *Tire Science and Technology*, TSTCA, Vol. 20, 1992, pp. 33-56.
7. Faria, L.O., Bass, J.M., Oden, J.T. and Becker, E.B., "A three-dimensional rolling contact model for a reinforced rubber tire," *Tire Science and Technology*, TSTCA, Vol. 17, 1989, pp. 217-233.
8. Padovan, J., "Finite element analysis of steady and transiently moving/rolling nonlinear viscoelastic structure," *Comp. and Struct.*, Vol. 27, 1987, pp.249-286.
9. Luchini, J.R., Peters, J.M. and Arthur, R.H., "Tire rolling loss computation with the finite element method," *Tire Science and Technology*, TSTCA, Vol. 22, 1994, pp. 206-222.
10. Sarkar, K., Kwon, Y.D. and Prevorsek, D.C., "A new approach for the thermomechanical analysis of tires by finite element method," *Tire Science and Technology*, TSTCA, Vol. 15, 1987, pp. 261.
11. Jeusette, J.P. and Theves, M., "Finite element analysis of tire/rim interface forces under braking and cornering loads," *Tire Science and Technology*, TSTCA, Vol. 20, 1992, pp. 83-105.
12. Tseng, N.T., Pelle, R.G. and Chang, J.P., "Finite element simulation of the tire/rim interface," *Tire Science and Technology*, TSTCA, Vol. 17, 1989, pp. 305-325.
13. Tseng, N.T., Pelle, R.G., Chang, J.P. and Warholic, T.C., "Finite element simulation of destructive tire testing," *Tire Science and Technology*, TSTCA, Vol. 19, 1991, pp. 2-22.
14. Murakoshi, H., Ide, H. and Nishihata, S., "An approach to vehicle pull using finite element model," *Tire Science and Technology*, TSTCA, Vol. 20, 1992, pp. 212-229.
15. Kao, B.G. and Muthukrishnan, M., "Tire transient analysis with an explicit finite element program," to be submitted to *Tire Science and Technology*.
16. Hallquist, J.O., *LS-DYNA3D Theoretical Manual*, LSTC Report 1018 Revision 3, pp. 16.39-16.41.
17. Belytschko, T. and Tsay, C.S., "Explicit algorithms for nonlinear dynamics of shells," ASME, AMD-Vol. 48, 1981, pp. 209-231.
18. Belytschko, T., Lin, J. and Tsay, C.S., " Explicit algorithms for nonlinear dynamics of shells," *Comp. Meth. Appl. Mech. Eng.*, Vol. 42, 1984, pp. 225-251.
19. Hallquist, J.O., *LS-DYNA3D Theoretical Manual*, LSTC Report 1018 Revision 3, pp. 6.1-6.12.
20. Schwerzler, D., "Tire Model Section," in *GM Total Vehicle System Modeling Course*, 1985.
21. Gardner, I. and Theves, M., "Computer modeling of a tire under cornering loads," *Tire Science and Technology*, TSTCA, Vol. 17, 1989, pp. 86-99.
22. Wong, J.Y., *Theory of Ground Vehicles*, John Wiley, 1993, Chapter 1, pp.3-76.
23. Pacejka, H.B. and Bakker, E., "The magic formula tyre model," in *Tyre Models for Vehicle Dynamics Analysis*, Supplement to *Vehicle System Dynamics*, Vol. 21, 1993, pp. 1-18.